NOTATION

T, temperature of plate or infinite prismatic body; Bi, Biot number; T_a, ambient temperature; F = W/λ; W, specific strength of energy sources; λ, thermal conductivity.

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MAIN TYPES OF CONJUGATE PROBLEMS IN HEAT AND MASS EXCHANGE*

N. I. Nikitenko

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The development of optimal technological processes and apparatuses for heat and mass exchange is placing more and more severe requirements on the suitability of mathematical models and the accuracy of their mechanization. Of great interest in this connection is the solution of problems in heat and mass exchange in the conjugate formulation, which makes it possible to take account more fully of the interrelation between the transfer processes taking place in the bodies which are in contact. In this case the description of the transfer processes at the interface between the phases makes use of boundary conditions of the fourth kind, which are differential equations that arise out of the laws of conservation of the relevant entities being transferred – energy, mass, momentum, etc.

Although boundary conditions of the fourth kind have been used for a fairly long time [1], the solution of problems in heat and mass exchange in the conjugate formulation was long restricted by the inadequate level of development of the analytic and numerical methods of solution. The number of published works devoted to the solution of conjugate problems (CP) in heat and mass exchange began to increase rapidly after the appearance of the works of Lykov and Perel'man [2, 3], which were the first to formulate an external CP in heat exchange and show the desirability of such a formulation. The rapid spread of investigations related to conjugate heat- and mass-exchange problems was greatly facilitated by the development of numerical methods of solution designed for use with computers.

The place occupied by CP in the theory of heat and mass exchange is similar in many ways to the position occupied by boundary-layer theory, ideal-liquid theory, mechanics of viscous continuous media, or

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statistical mechanics of liquids in fluid mechanics. Therefore it seems expedient to call the branch of the theory of heat and mass exchange which deals with CP "the theory of conjugate heat and mass exchange." Like the aforementioned areas of fluid mechanics, this theory requires not only the construction of mathematical models and the development of effective methods of solution but also a study of the question of the classes of processes for which its utilization is very appropriate.

Despite the large number of published works devoted to CP in heat and mass exchange, up to now there is still no classification of these problems, and there is not even any clear agreement as to what problems in heat and mass exchange should be regarded as conjugate problems. Thus, for example, [4] applies the term CP to transfer problems described by equations of various structure or type with some additional conditions at the interfaces between the different media to represent mathematically the laws of conservation of mass, energy, and momentum. We can scarcely agree with such a treatment of CP. From the aforementioned formulation, in particular, it would follow that the problem of heat exchange between two jets is not a conjugate problem, whereas a particular case of this – the problem of heat exchange between a jet and a solid – is a conjugate problem.

The following formulation seems more acceptable. Conjugate problems in heat and mass exchange are problems in the determination of fields of physical quantities (temperature, concentration, velocity, etc.) in a system of bodies possessing different properties (in other words, a heterogeneous system) when the laws of conservation of the relevant entities are used to find the values of these physical quantities at the boundaries between the bodies in the system. Mathematically, conjugate problems are problems related to the solution of a system of differential or integrodifferential equations which differ either in the values of the coefficients by some finite quantity or in their structure, when in order to find the desired functions on the common boundaries between the domains of definition of the indicated equations we must use some algebraic and differential relations. These relations can usually be represented in the form

$$\psi(P+0) = f_1\left(\psi(P-0)\right),$$

$$\frac{\partial\psi(P+0)}{\partial n} = f_2\left(\frac{\partial\psi(P-0)}{\partial n},\ \psi(P+0),\ \psi(P-0),\ V_n(P)\right),$$

where $V_n(P)$ is that component of the velocity of motion of a point P lying on the boundary surface which is in the direction of the normal n to this surface. Like other problems connected with the study of physical fields, CP may be stationary or nonstationary, one-dimensional or multidimensional, simply connected or multiply connected. Depending on the physical state of the heterogeneous system, CP may be subdivided into three classes. The first class includes the CP of molecular transfer. These are related to the study of the interaction between solids and can be further subdivided into the following main types:

1) CP in heat conduction, in which the desired function is a temperature function T for a heterogeneous system;

2) CP in diffusion, in which we are trying to find the concentration C of a component;

3) CP in radiant transfer, in which we determine the radiated energy density function u in a system of absorbing solids;

4) CP in thermoelasticity, involving the search for the function T and a function characterizing the deformation of a system – for example, the displacement vector U;

5) CP in mass elasticity (the desired functions are C and U).

It should be noted that CP in heat conduction and diffusion can be mathematically described in the same way if the boundaries separating the bodies of the system are motionless.

The second class includes CP in convective transfer, connected with the study of the interaction between liquid media. The main types of CP in this class are the following:

1) CP in heat transfer (the desired function is T);

2) CP in mass transfer (the desired function is C);

3) CP in flow and heat transfer (the desired functions are the velocity vector function V and the temperature T);

4) CP in flow and mass transfer (the desired functions are V and C);

5) CP in radiant-convective transfer (the desired functions are u and V).

The third class includes CP in molecular-convective transfer, related to the investigation of the interaction between solids and liquids. The main types of CP in this class are:

1) CP in heat transfer (the desired function is T);

2) CP in mass transfer (the desired function is C);

3) CP in flow and heat transfer (the desired functions are V and T);

4) CP in flow and mass transfer (the desired functions are V and C);

5) CP in flow and heat transfer for deformed systems (the desired functions are V, T, and U);

6) CP in flow and mass transfer for deformed systems (the desired functions are V, C, and U).

In addition to the main types, each of the classes of CP also has types which are combinations of the main types – for example, CP in heat and mass transfer (the desired functions are T and C for a system of solids and liquids) or thermomassoelasticity [5] (the desired functions are T, C, and U for a system of solids).

The mathematical models corresponding to CP are extremely diverse. In formulating CP of the first kind, the fundamental equations used may be Fourier's heat-conduction equations, hyperbolic transfer equations [1] or integrodifferential transfer equations [5]. In formulating CP of the second and third classes, we may determine the function V by means of Navier–Stokes equations, boundary-layer equations, Euler's equations, etc.

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